Correlation measurements in a non-frozen pattern of turbulence

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The properties of a turbulent flow are often described in terms of velocity correlations in space, in time, and in space-time. In this paper the interpretation of velocity correlation measurements which are made in a region of high-intensity turbulence is considered in some detail. Under these conditions it is shown that some account must be taken of the effects of both mean and fluctuating shear stresses which are continuously modifying the turbulent structure. For an almost frozen pattern, for example, in the turbulence behind a grid, the turbulent convection velocity is amost equal to the mean flow velocity, while the space correlation and auto-correlation of the velocity fluctuations are simply related through this velocity. In contrast to this, when the intensity is high, the convection velocity may differ considerably from the mean velocity, while it is shown that different turbulent spectral components appear to travel at different speeds. This means that the turbulent spectrum and the turbulent space scales are no longer simply related. For example, the high-frequency spectral components may be ascribed to both the high-velocity eddies and the small wave-number components acting together.

Experimental results are presented which indicate the conditions under which the assumption of a frozen pattern leads to uncertainties in the subsequent interpretation of the measurements. The measurements also show that the observed difference between the mean and the convection velocity may be qualitatively explained in terms of the skewness of the velocity signals.

1. Introduction

The investigation of various types of turbulent flow has received considerable impetus during the last decade because of the importance of the pressure fluctuations associated with these flows. The fluctuations of greatest practical interest at the present time are those associated with turbulent shear flows, in particular the turbulent boundary layer and the turbulent jet.

The use of correlation techniques to examine the statistical properties of turbulent velocity and pressure fluctuations is now well known. One may cite the experiments of Townsend (1947) on isotropic turbulence, the investigations of the turbulent boundary layer due to Favre, Gaviglio & Dumas (1957, 1958) and Willmarth (1959) and the investigations of jet turbulence due to Laurence (1956) and Davies & Fisher (1963), to mention but a few. However, some care is necessary in the interpretation of correlation measurements in the presence of a mean shear and an appreciable turbulence intensity. This is due to the fact that turbulence is being continuously created and modified by the shear stresses. Thus Taylor's hypothesis (Taylor 1938), that turbulence may be regarded as a frozen pattern of eddies being swept past the observer, is no longer strictly valid. From a consideration of the terms in the Navier–Stokes equations, Lin (1952) has in fact shown that Taylor's hypothesis is valid only if the turbulence level is low, viscous forces are negligible and the mean shear is small. The aim of the present paper is to discuss briefly the information that can be obtained from correlation measurements and particularly to examine their interpretation for both a frozen and non-frozen pattern of turbulence. In addition an experiment carried out in the mixing region of a subsonic jet is described in which the correlation properties, in particular the convection velocity and the time scale, have been measured as a function of the frequency of the turbulence. The spectrum function of the temporal fluctuations associated with various components of the turbulence is also estimated.

The results indicate clearly that the detailed interpretation of correlation measurements in regions of high shear and appreciable turbulent intensity is subject to large uncertainties. The convection velocity of the velocity fluctuations is found to be frequency dependent, increasing with increased frequency. This suggests that to some extent the higher frequencies may be due to high velocity components rather than large wave-number eddies as would be suggested by a constant value of the convection velocity. Further difficulties, arising from the asymmetric distribution of the velocity signal about its mean (Davies & Fisher 1963) are discussed in detail in §§2.5 and 3. The spectra of the temporal fluctuations associated with the various turbulence components (see figure 7 and §2.4) indicate that the associated energy is distributed over a considerable frequency range. Evaluation of these frequencies from the experimental results lends strong evidence to the hypothesis of §1.3 that these temporal fluctuations are strongly associated with a range of eddy velocities around the convection velocity.

The observed differences between the convection velocity and mean velocity of the flow, as observed in the mixing region of a jet, is discussed in §2.5. It is shown that this difference can be due to the skewness of the probability density diagram of the velocity fluctuations, the convection velocity being higher than the mean velocity for positive skewness, the converse being true for regions of negative skewness.

1.1. *Correlation techniques*

The correlation coefficients most commonly measured are the space correlation, the auto-correlation and the cross-correlation, the former two being merely special cases of the more general cross-correlation coefficient. Cross-correlation measurements involve the correlation of signals from two spatially separated measuring positions, the signal from the upstream point being delayed by a time τ. Thus if $v_1(0, 0)$ denotes the signal received at one point at time $t = 0$ and

$v_2(x, \tau)$ denotes the signal received at a point distance x from the first at time $t = \tau$ their cross-correlation coefficient $R(x, \tau)$ may be defined as

$$R(x, \tau) = \frac{\overline{v_1(0, 0)\, v_2(x, \tau)}}{(\overline{v_1(0, 0)^2})^{\frac{1}{2}}\, (\overline{v_2(x, \tau)^2})^{\frac{1}{2}}},$$

where the over-bar denotes an average over a period of time sufficiently long to obtain stationary values.

The space correlation, which involves the comparison of the instantaneous signal received at two spatially separated measuring points is thus merely the cross-correlation for zero time delay, i.e. $R(x, 0)$, whilst the auto-correlation coefficient, which involves the correlation of the signal received at a measuring point with the signal received at the same point a time τ previously is the cross-correlation for zero separation of the measuring points, i.e. $R(0, \tau)$. We may also note in passing that the auto-correlation curve is the Fourier transform of the power spectrum of the turbulence. Finally, we may deduce from the inverse spreading relation between Fourier transform pairs that if the auto-correlation curve is a slowly decreasing function the turbulent spectrum will comprise a narrow band of frequencies, the converse also being true.

1.2. *The frozen pattern*

On the assumption of Taylor's hypothesis (see § 1) we see that a signal received at one measuring position will be received at a second position, distance x_1 directly downstream from the first, at a time, τ_1, later. Thus the value of the particular cross-correlation coefficient $R(x_1, \tau_1)$ will have a maximum value of unity. We may thus define the velocity of convection, U_c, of the pattern as the x_1/τ_1.

Since $R(x_1, \tau_1)$ represents the maximum value of the cross-correlation curve for the measuring-point separation x_1 we may alternatively define the convection velocity as the ratio x_1/τ for which $\partial R(x_1, \tau)/\partial \tau = 0$. Alternatively we can consider a fixed value of the time delay and vary the measuring-point separation. An identical velocity is then defined as the ratio x/τ_1 for which $\partial R(x, \tau_1)/\partial x = 0$. However, it is important to realize that it is only when the temporal rate of change of the turbulent pattern is zero that these two definitions are equivalent.

It is also obvious that the auto- and space-correlation coefficients are exactly related, for such a frozen pattern, through the convection velocity by the relation

$$R(\tau) = R(x = U_c \tau).$$

Further we may deduce from the frozen nature of the pattern that all eddies travel with this unique velocity and hence the mean velocity and convection velocity are identical.

Let us finally consider the meaning and interpretation of the energy spectrum of a frozen pattern of turbulence. Consider a series of cross-correlation measurements performed at a number of measuring-point separations x_i ($i = 1, 2, 3, \ldots$, etc.). Each cross-correlation curve will pass through a maximum value unity at a value of the time delay τ_i where $\tau_i = x_i/U_c$.

The auto-correlation in a frame of reference moving with the convection velocity will thus be the line $R(x-U_c\tau, \tau) = 1$. Fourier transformation of this moving-axes auto-correlation will yield a spectrum in which the power is contained in an infinitely narrow band of frequencies centred on zero frequency. Thus an observer travelling with the convection velocity is aware only of turbulence components of zero frequency. Hence the spectrum observed in the stationary frame of reference is not due to any temporal turbulence fluctuations, but due to eddies being convected past the observer at the convection velocity. Thus any fixed-frame frequency f is related to the spatial extent, λ, of the eddy producing it by a relation of the form

$$f = U_c/\lambda.$$

This may alternatively be written

$$\omega = 2\pi f = KU_c,$$

where K denotes the wave-number of the eddy. Hence to summarize for a frozen pattern we see that:

(*a*) The convection velocity can be measured from a knowledge of the time delay which will maximize the cross-correlation curve for a particular measuring point separation. In addition this velocity and the mean velocity of the flow are identical.

(*b*) The space- and auto-correlation curves are simply related through this velocity.

(*c*) A knowledge of the turbulence spectrum and the convection velocity permits the measurement of the spatial extent of the component eddies.

1.3. *The non-frozen pattern*

Let us next consider the situation which is encountered in practice with both the turbulent boundary layer and the turbulent jet. Here the presence of shearing forces cause the convected pattern to change as it travels downstream. The result of a series of cross-correlation measurements of such a pattern is shown in figure 1. It can be seen that although each cross-correlation curve rises to a maximum at some value of the time delay, clearly indicating the presence of convection, the amplitude of this maximum is a function of the measuring-point separation. In addition the convection velocity is no longer defined by the time delay at which the maximum of a particular cross-correlation curve occurs, but by the time delay at which the envelope of all the cross-correlation curves intersects the curve for a particular measuring point separation. For consider an observer travelling at the velocity so defined. The auto-correlation in his frame of reference is the envelope of the cross-correlation curves and geometric consideration of figure 1 shows that this is the auto-correlation curve having the maximum time scale, the time scale being defined as that time delay for which the moving axis auto-correlation falls to a value $1/e$. Thus it is in this frame of reference that the temporal rate of change of the turbulence is a minimum. It thus seems physically acceptable that it is this velocity which defines the mean rate of transport of the energy bearing eddies, i.e. the convection velocity. Further at any particular instant of space and time the eddy pattern is changing at a rate given by $dR(x, \tau)/d\tau$. Thus, if the observer is travelling with the eddy velocity, the auto-correlation in his frame of reference should also be changing at this same rate. Thus for any distance downstream the auto-correlation in his frame of reference should be a tangent to the fixed axis cross-correlation curve for that distance, i.e. the envelope of the cross-correlation curves as we have suggested. One further point in favour of this definition of convection velocity can be seen from consideration of the

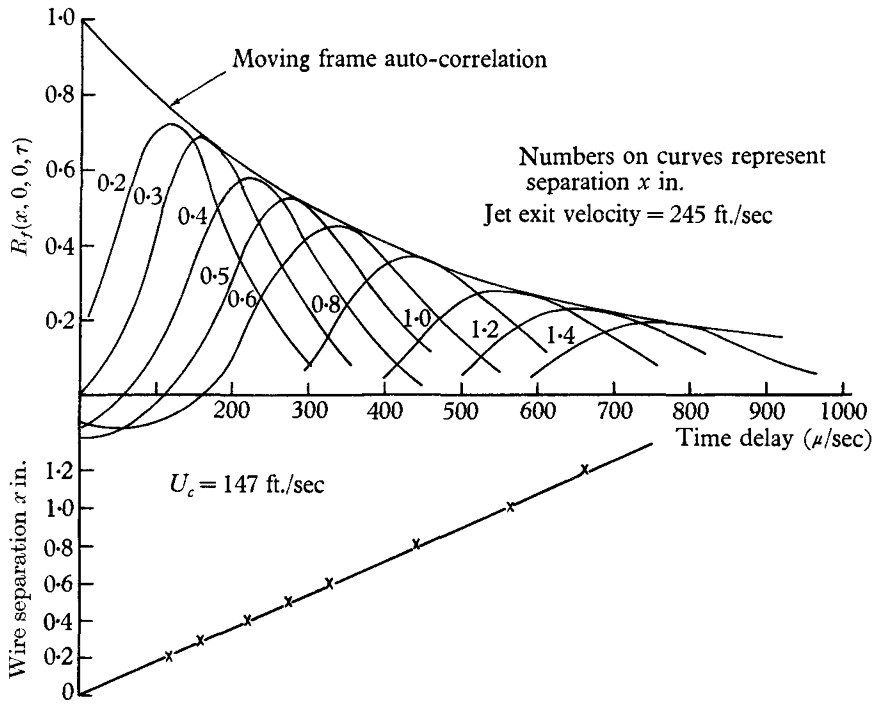

FIGURE 1. Space-time correlation of axial velocity fluctuations (downstream separation). Fixed wires at $X/D = 1{\cdot}5$, $Y/D = 0{\cdot}5$.

turbulence spectrum observed in this frame of reference. We have already noted that, for a frozen pattern of turbulence, the effect of convection is to broaden the band of observed frequencies. Although for an unfrozen pattern of turbulence a band of truly temporal, turbulent frequencies will exist, any convection of the turbulence relative to the observer will similarly broaden the band of observed frequencies. Thus a good criterion for the definition of the moving-axis auto-correlation in a frame of reference moving with the convection velocity might well be that auto-correlation which, when transformed, will yield the narrowest band of temporal frequencies. From the inverse spreading relationship between Fourier-transform pairs, mentioned previously, we see that this is the auto-correlation of maximum time scale as suggested above.†

† Since the initial preparation of this paper the problem of defining a meaningful convection velocity in turbulent shear flows has been discussed by Wills (1963). It is suggested that a meaningful definition of the convection velocity is the ratio x/τ (δ/τ in Wills's notation) at the point where $\partial R(x,\tau)/\partial x = 0$. In the definition suggested above we have demanded

Certain difficulties now arise, however, in obtaining turbulence scales from spectrum measurements in a fixed frame of reference. The existence of a finite moving-axis time scale indicates that turbulence frequencies arising from temporal fluctuations are present as well as those which are due purely to convection as encountered in the frozen pattern situation. It is of some interest to what follows to discuss briefly the possible origins of these temporal fluctuations. It is often convenient to assume that all components of the turbulent pattern travel downstream with the convection velocity. In this case the temporal frequencies must be due to the distortion of the component eddies as they are acted upon by the mean shear. However, it is felt that this concept is an over-simplification of the true situation. The results of experiments to examine the fluctuation of turbulence velocity signals about their mean (Davies & Fisher 1963) indicate that eddies travelling with a wide range of velocities are present in, for example, the shear layer of a subsonic jet. Consider now an eddy of wave-number K travelling with a velocity U. To a stationary observer this eddy appears as a component of frequency $w_f = KU$, whereas to an observer in a frame of reference travelling with the convection velocity it appears as a component of frequency $w_m = K|U-U_c|$. Thus it is apparent that even neglecting the distortion of the component eddies as they travel downstream, non-zero frequency components can be observed in the moving frame due to a range of eddy velocities about the convection velocity. It is by no means clear at present which of these two effects, if either, is dominant in producing the turbulence spectrum observed in the moving frame of reference. Indeed one might well expect an eddy travelling at a velocity far removed from the mean velocity of the pattern to interact violently with neighbouring eddies thus becoming rapidly distorted. To this extent the two effects might well be mutually reinforcing.

It is interesting to note that a mutual uncertainty must always exist in the presence of shear forces with regard to the respective values of the wave-number and velocity although their product may be known exactly. A single-point measurement can be used, theoretically at least, to yield the frequency of a component eddy giving the instantaneous product $w = KU$. However, this single measurement yields no information about the eddy velocity. However, if now the eddy is permitted to travel a finite distance in order that its velocity may be measured it may become distorted by body forces. Thus although its velocity is now known exactly an uncertainty exists in the wave-number K.

that the temporal rate of change of the auto-correlation, in a frame of reference moving with the convection velocity, shall be equal to the temporal rate of change of the cross-correlation curve at that point. In any arbitrary frame of reference the apparent rate of change of the pattern will be the algebraic sum of the spatial and temporal rates of change. Thus Wills's condition that the spatial rate of change shall be zero in a frame of reference moving with the convection velocity is satisfied and the choice between the two definitions is seen to be merely one of experimental convenience. However, it is important to notice that a velocity based on the ratio x/τ at the point where $\partial R(x,\tau)/\partial\tau = 0$ does not constitute an equivalent definition, as Wills points out. In addition very little physical significance can be attached to such a definition as it merely represents the velocity of a frame of reference in which, at the point considered, the temporal and spatial rates of change of the pattern are of equal magnitude, but of opposite sign.

Let us now return to the problem of obtaining turbulence scales from spectrum measurements. A stationary observer while observing an eddy of apparent frequency w has, *a priori*, no method of deciding whether this frequency is due purely to the passage of the eddy past him at the convection velocity or whether it is due to the combination of convection and temporal fluctuations. Thus it is only when the temporal frequencies are small compared with those due to convection that a reasonable estimate of the wave-number spectrum can be obtained from the measurement of a frequency spectrum.

It is also apparent that the space- and auto-correlation curves are no longer exactly related in an unfrozen pattern of turbulence. The distortion of the pattern as it is convected between two measuring positions, distance x apart, causes the auto-correlation $R(\tau)$ (where $\tau = x/U_c$) to have a value less than the space correlation $R(x)$.

A knowledge of the moving-axis time scale and the shape of the moving axis auto-correlation will still permit the transformation between the space and auto-correlation to be performed, but having obtained this information sufficient experimental data is usually available to measure either or both directly. It should perhaps be mentioned that although the transformation cannot be performed exactly in an unfrozen pattern of turbulence it often happens that the moving-axis time scale is sufficiently long to permit a reasonable approximation to be obtained. However, it cannot be too strongly emphasized that care should be taken to investigate the degree of approximation involved and in particular to ensure that the correct value of the convection velocity is used. Further, previous experiments (Davies, Fisher & Barratt 1963) indicate that the variation of convection velocity across the shear layer of a subsonic jet bears little resemblance to the variation of the mean velocity, the former varying far less rapidly. The mean velocity should therefore never be used in place of the convection velocity.

To summarize for a non-frozen pattern we see that:

(*a*) The convection velocity can be measured from a knowledge of the value of the time delay at which the envelope of a series of cross-correlation curves touches that curve corresponding to a given measuring-point separation. The convection velocity and the mean velocity are not identical in this situation.

(*b*) The space- and auto-correlation curves are not exactly related and attempts at a transformation of the type suggested for a frozen pattern leads to an underestimate of the space correlation.

(*c*) The turbulent spectra and the scale of turbulence are no longer simply related.

To relate the frequency spectrum of a non-frozen pattern of turbulence to its wave-number spectrum the following features require investigation. First, it must be ascertained whether or not various wave-number components are convected downstream at the same velocity. Secondly, the change of apparent frequency of a particular wave-number component when observed in a fixed frame of reference, due to any purely temporal fluctuations must be estimated. Hence, ideally, we require to isolate various wave-number components and then to perform cross-correlation measurements on these components. Thus the convection velocity could be measured as a function of eddy size and transformation of the associated moving-axes auto-correlations would yield the spectrum of the temporal fluctuations associated with these various components. However, techniques for the isolation of particular wave-number components are not at present available. In the present experiment, described below, various fixed-axes frequency components have been isolated and the convection velocity measured as a function of apparent frequency. In addition the spectrum of temporal fluctuations associated with these various components has been obtained by transformation of their moving-axes auto-correlations. However, it should be carefully noted that this experiment, although the best available approximation to the ideal experiment mentioned above, differs from it since each fixed-axes frequency component comprises a band of wave-number components and an associated band of temporal fluctuations. It is only when the frequencies associated with the temporal fluctuations are small compared with frequencies observed in the fixed frame of reference that the moving-axes spectrum can be considered to be that associated with a particular wave-number component.

1.4. *The amplitude of filtered correlations*

A previous experiment, involving the correlation of filtered wall pressure fluctuations of a turbulent boundary layer, has been performed by Harrison (1958). The results indicate that if the pressure fluctuations, as measured at two points, one directly upstream of the other, are first filtered and a space correlation then performed on the filtered signals, the amplitude of the correlation coefficient is a unique function of the Strouhal number fx/U_c. The attractions of such a result, if it could be shown to be generally applicable to turbulent fluctuations in a shear layer over a wide range of frequencies, are apparent, since the measurements of the space correlation curve at one frequency would permit the calculation of similar curves at all other frequencies. However, the result contains, as we shall see below, an anomaly suggesting that low-frequency components must remain correlated over very considerable distances. It was therefore decided to investigate in some detail the range of applicability of this result when applied to axial velocity fluctuations in the mixing region of a jet.

Let us first consider the implications of Harrison's result. Suppose we have a completely frozen pattern of turbulence, being convected between two measuring positions. If we now select an infinitely narrow band of frequencies from both signals and measure the cross-correlation in this narrow band, we find the correlation curve obtains the value unity at a time delay τ where $\tau = x/U_c$ and then follows a cosine law, with the periodicity of the filter about this point. Thus,

$$R_f(x, \tau) = \cos 2\pi f(\tau - x/U_c)$$

and the filtered space correlation is therefore

$$R_f(x, 0) = \cos(2\pi fx/U_c). \qquad (1)$$

Thus, for a completely frozen pattern of turbulence we would expect the value of the filtered space correlation to be a unique function of the Strouhal number as shown by equation (1).

Let us next consider the analogous case when the pattern is not frozen. The cross-correlation will rise to a maximum at a value of time delay given approximately by $\tau = x/U_c$ and thence vary according to the cosine law as above. However, this maximum value will no longer be unity but will have some value A less than unity. Thus

$$R_f(x, 0) = A\cos(2\pi fx/U_c). \tag{2}$$

If now the filtered space correlation is to be a function of Strouhal number alone, as has been implied, then equation (2) suggests A must similarly be a function of Strouhal number only. Since A represents closely the maximum amplitude to which a given cross-correlation curve will rise, if it is to be a function of fx/U_c alone, it is apparent that low-frequency components must have a moving-axis auto-correlation which decreases extremely slowly in time compared with the higher-frequency components. We are thus led to the anomalous result that low-frequency components are very little effected by shear.

In the present experiment the necessary correlations have been performed on the axial velocity fluctuations to investigate the dependence of filtered space correlations on Strouhal number, over a useful range of frequencies. It is found, unlike Harrison's results, that the amplitude is not a function of Strouhal number alone.

2. The experiments

The velocity fluctuations have been measured using the constant-temperature, hot-wire anemometer system previously described by Davies *et al.* (1963); Davies & Fisher (1963). The jet is 1 in. in diameter and in the present experiment the stationary probe was positioned 1·5 in. downstream from the lip, half a diameter from the jet axis. The second probe was similarly positioned and then moved to successive positions downstream of the first on a line parallel to the jet axis. This location was chosen for the experiment as it was known from previous experiments that the moving-axis time scale is shorter here than farther downstream. Since the unique variation of the filtered space correlation with Strouhal number and the exact relationship between turbulent spectra and scales has been established for a frozen pattern (i.e. infinite time scale) it was obviously desirable to work in a region of the jet where the time scale is short.

The signal from the hot-wire set consists of a d.c. level corresponding to the mean velocity, with an a.c. component, representing the turbulent fluctuations, superimposed. The d.c. level was removed using a blocking condenser and thence after suitable attenuation the signal was recorded on the F.M. system of an Ampex tape recorder. Some care was taken to ensure that the capacitor–resistor circuit formed by the blocking condenser and attenuator had a flat response over the frequency range of interest. The high signal-to-noise ratio ($\simeq 50$ db) available using a frequency modulated recording system is desirable, particularly when comparing correlations of components in a falling part of the energy spectrum, as will be seen later.

The signals from both hot wires were recorded simultaneously on two tape-recorder channels which had been previously checked for phase differences. For the purpose of correlation the signals were played back through two matched Bruel and Kjaer $\frac{1}{3}$-octave filters to the Southampton correlator. A time delay unit was also used to perform cross-correlations. Both the correlator and time delay unit have been described in detail by Allcock, Tanner & McLachlan (1962).

2.1. *Results*

Before proceeding with the filtered correlations the cross-correlation of the overall signals were measured. The results, plotted as a function of time delay, are shown in figure 1. Each curve corresponds to a different measuring-point separation. The separations used are indicated on the diagram. The envelope of these curves, the auto-correlation in the moving frame of reference, is also shown. The convection velocity, measured by plotting the wire separation against the time delay at which the envelope touched the cross-correlation curve was found to be $0{\cdot}61\ U_0$ where U_0 denotes the jet exit velocity.

The turbulence signals were next filtered into $\frac{1}{3}$-octave bands at centre frequencies of 500, 800, 1250, 2500, 4000 and 5000 c/s and the cross-correlations of these signals measured. Three typical results are shown in figures 2(*a*), (*b*) and (*c*). The result of plotting the value of the correlation coefficient at the intersection of the moving axes auto-correlation curve and the cross-correlation curve against the corresponding hot-wire separation is shown figure 3. This shows that the rate of decrease of optimum correlation with distance is to a first approximation independent of frequency over the range 500–1250 c/s, but a more rapid decrease is apparent for the higher-frequency components.

The convection velocity has also been measured for the various frequency components in the manner described during the discussion of the overall signals. The results, shown in figure 4, show a definite increase of convection velocity with frequency from $0{\cdot}45U_0$ at 500 c/s to $0{\cdot}68U_0$ at 5 kc/s. It is also apparent from comparison of figures 2(*a*), (*b*) and (*c*) that the moving-axis time scale decreases as the frequency is increased above 1250 c/s. The variation of this scale is shown in figure 5.

2.2. *Discussion of the results*

Before proceeding to examine the variation of the filtered space correlation with Strouhal number two precautions which are necessary if erroneous results are not to be obtained will be discussed.

(*a*) *The effect of finite bandwidth.* The first involves the necessity in practice of using a finite bandwidth when filtering the signals. This means that the cross-correlation curve, instead of being an undamped cosine wave as suggested for an infinitely narrow filter band, is damped on either side of its main peak, which will occur close to $\tau = x/U_c$. If therefore the curve passes through several cycles between $\tau = x/U_c$ and $\tau = 0$ the net effect is an underestimate of the filtered space correlation. This effect will obviously be more prevalent at higher frequencies and will therefore cause these values of the space correlation to be particularly underestimated, a fact, which as we shall see later might well

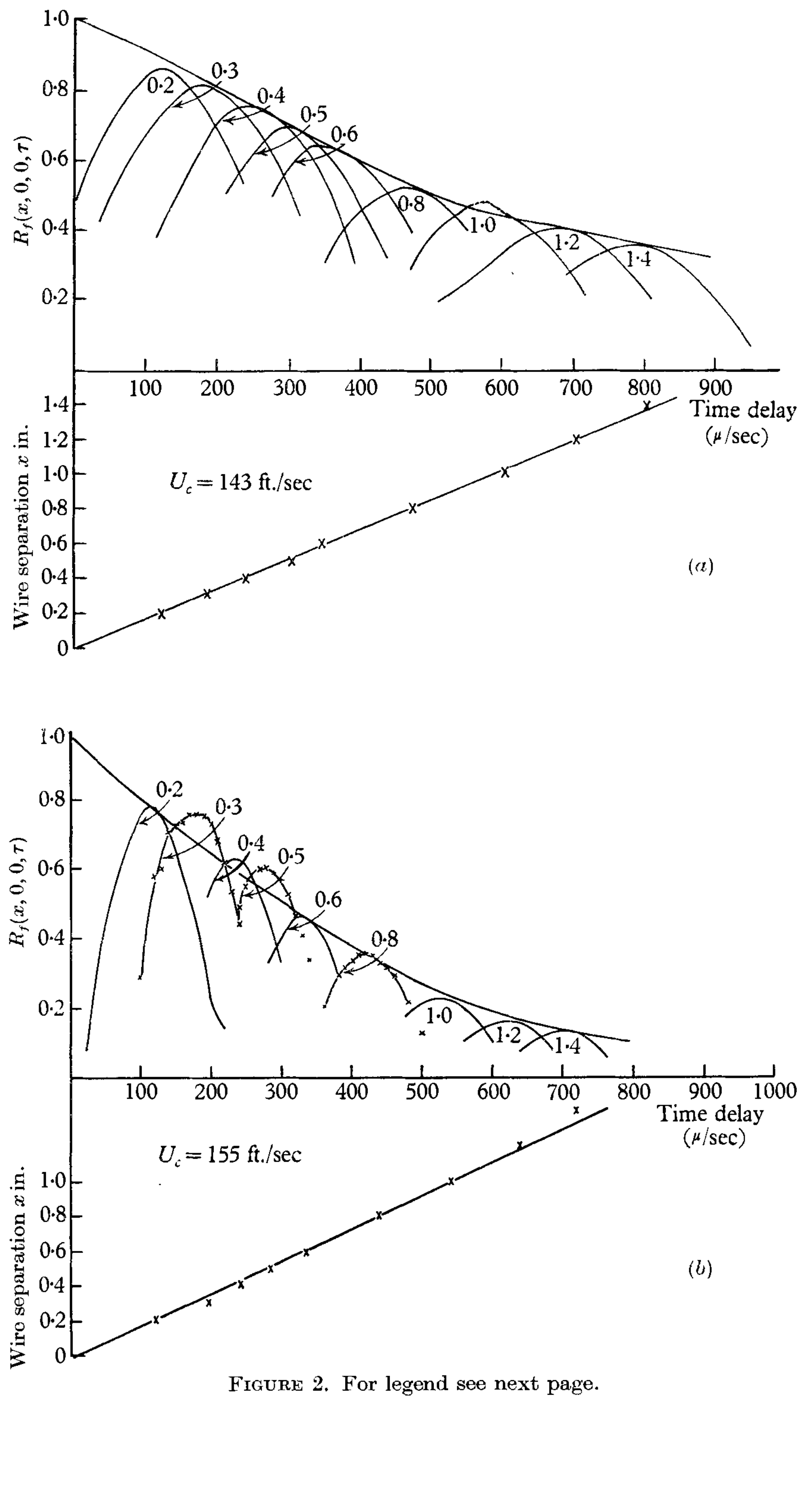

FIGURE 2. For legend see next page.

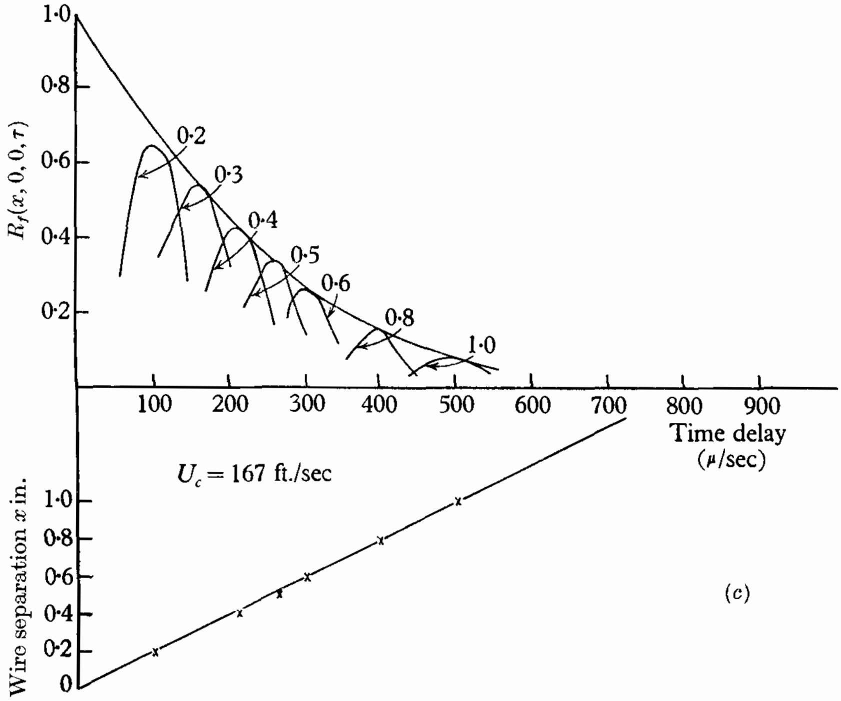

Figure 2. Space-time correlations of axial velocity fluctuations filtered in $\frac{1}{3}$-octave bands. (*a*) Centre band frequency = 1·25 kc/s; (*b*) centre band frequency = 2·5 kc/s; (*c*) centre band frequency = 4 kc/s.

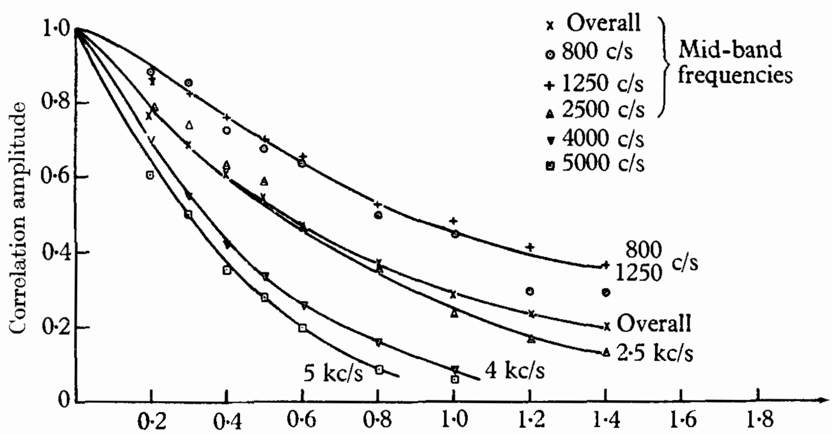

Figure 3. Rate of decrease of optimum correlation *vs* wire separation for various frequencies.

suggest a unique dependence of the space correlation on Strouhal number at least for the higher frequency components (see figure 6). In the present series of experiments the correlation has been measured over the complete range of time delays from $\tau = 0$ to $\tau > x/U_c$ so that an estimate of this effect can be made and allowed for in the final value of the space correlation.

(*b*) *The effect of electronic noise.* Suppose we wish to correlate two signals v_1 and v_2. Then their true correlation coefficient R_T is

$$R_T = \frac{\overline{v_1 v_2}}{(\overline{v_1^2})^{\frac{1}{2}}\,(\overline{v_2^2})^{\frac{1}{2}}}.$$

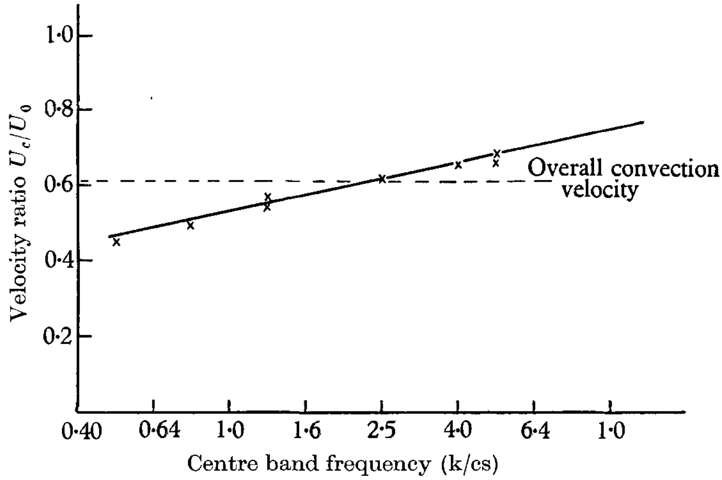

FIGURE 4. Variation of convection velocity with filter frequency.

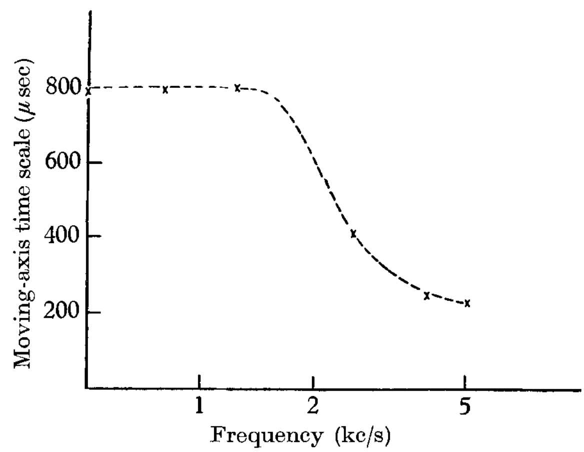

FIGURE 5. Variation of moving-axis time scale with centre-band frequency.

However, suppose superimposed on these signals we have a certain amount of noise introduced by the electronic systems through which the signals have been passed. Let these be a and b on signals v_1 and v_2, respectively. Then the apparent correlation coefficient R_A is

$$R_A = \frac{\overline{(v_1+a)\,(v_2+b)}}{(\overline{(v_1+a)^2})^{\frac{1}{2}}\,(\overline{(v_2+b)^2})^{\frac{1}{2}}} = \frac{\overline{v_1 v_2}}{(\overline{v_1^2}+\overline{a^2})^{\frac{1}{2}}\,(\overline{v_2^2}+\overline{b^2})^{\frac{1}{2}}}.$$

Since the signal and noise and the two noise signals are mutually random, $\overline{v_1 a} = \overline{v_2 a} = \overline{v_1 b} = \overline{v_2 b} = \overline{ab} = 0$. Thus if we write

$$\overline{a^2}/\overline{v_1^2} = E_1 \quad \text{and} \quad \overline{b^2}/\overline{v_2^2} = E_2,$$

we find the ratio of the true correlation coefficient to the apparent value is

$$R_T/R_A = \{(1+E_1)(1+E_2)\}^{\frac{1}{2}}.$$

Thus it is apparent that the effect of electronic noise is to reduce to correlation coefficient by a factor which depends on the mean-square signal-to-noise ratios. Precautions to ensure that this effect does not appreciably alter results are particularly important when performing correlations in frequency bands over

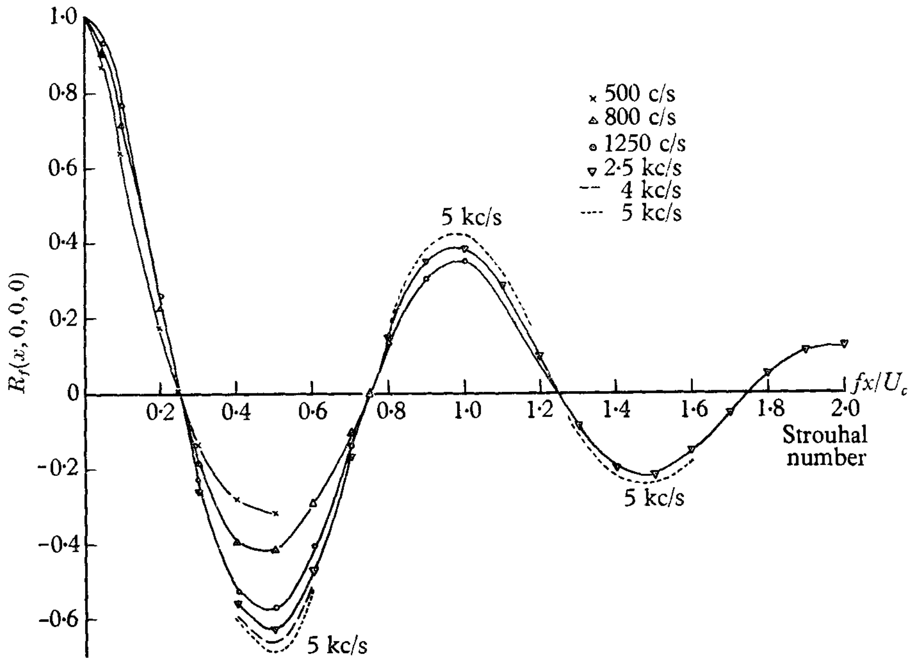

FIGURE 6. Amplitude of filtered space correlations.

a range of frequencies where the spectral density may decrease appreciably, thus increasing the factors E_1 and E_2 if it is assumed that the electronic noise is evenly distributed over all frequencies.

In the present experiment the turbulence spectrum function can be considered to be flat up to some frequency f_c and then to decrease at 6 db per octave beyond this point. For the stationary upstream probe f_c was approximately 1600 c/s, whilst for the downstream probe the value was somewhat lower depending on its position. Returning now to figure 3 we see it is for components of frequency greater than 1250 c/s that the measured correlation coefficients begin to decrease more rapidly, suggesting at first sight that this is due to a decrease of the signal-to-noise ratio as the spectral density decreases. However, calculations of the noise levels necessary, if the decrease of moving-axes time scale with increase of frequency were due entirely to this effect, indicate not only are these levels far in excess of those present, but in addition an increase of the actual level in one or both channels would be necessary as the probe separation is increased if the effect were to be entirely eliminated on this basis. Thus it seems fair to conclude that in fact the decrease of the moving-axes time scale above the frequency f_c is a real effect, although the exact extent to which it may be amplified by noise is not entirely certain.

2.3. *The dependence of the filtered space correlations on Strouhal number*

We have previously noted in equation (2) that the filtered space correlation is given by

$$R_f(x, 0) = A\cos(2\pi fx/U_c).$$

Thus we can calculate the value of $R_f(x, 0)$ for any value of the Strouhal number if A is known. For a particular frequency and convection velocity the value of x corresponding to a Strouhal number can be found and A can then be obtained from figure 3. The result of computing the filtered space correlation for the various frequency bands is shown in figure 6. It is apparent that although the curves are damped cosine curves, as expected, the amplituded increases with increased frequency throughout the range of frequencies investigated. The difference is particularly noticeable near the peaks of the curves. It is felt that this method of computing the variation is more satisfactory than the plotting of isolated experimental values since in the latter case only a few points in the region of the peaks may be available for comparison leading to erroneous conclusions. However, the comparison of the curves with direct measurement of the space correlation agree very closely once the latter have been corrected for bandwidth damping. It must be concluded therefore that the filtered space correlation is not a function of Strouhal number alone. In fact the results of figure 3 indicate that below the spectrum break point the value of A in equation (2) is independent of frequency and although a decrease is apparent above this value it is not sufficiently rapid to yield a Strouhal number dependence.

2.4. *Spectra in the moving frame*

We have previously noted in §1.1 that the spectrum of the turbulence, as observed in the frame of reference travelling with the convection velocity, can be obtained by transformation of the auto-correlation in this frame. Denoting this auto-correlation by $R(x-U_c\tau, \tau)$ the non-dimensional spectrum function is

$$W(f) = 4\int_0^\infty R(x-U_c\tau, \tau)\cos\omega\tau\, d\tau.$$

If further we assume the moving-axes auto-correlation to be of the form

$$R(x-U_c\tau, \tau) = e^{-K\tau},$$

where K is obviously the reciprocal of the moving-axes time scale, the spectrum function is

$$W(f) = 4K/(\omega^2+K^2). \tag{3}$$

It should be noted here that although the assumed form for the auto-correlation shows good agreement with the experimentally determined form (viz. figures 3 (*a*), (*b*) and (*c*)) over a large portion of the time delays of interest it is expected that in the limit of zero time delay the auto-correlation coefficient will approach the value unity asymptotically rather than with the finite gradient $-K$, as suggested by the present expression. The net effect of this would be to cause the spectrum function to decrease at the higher frequencies rather more quickly than is suggested by (3).

The spectrum functions for the 1250 and 5000 c/s fixed-axes components, as seen in frames of reference moving with their respective convection velocities, are shown in figure 7. It can be seen that, unlike the frozen pattern which, when

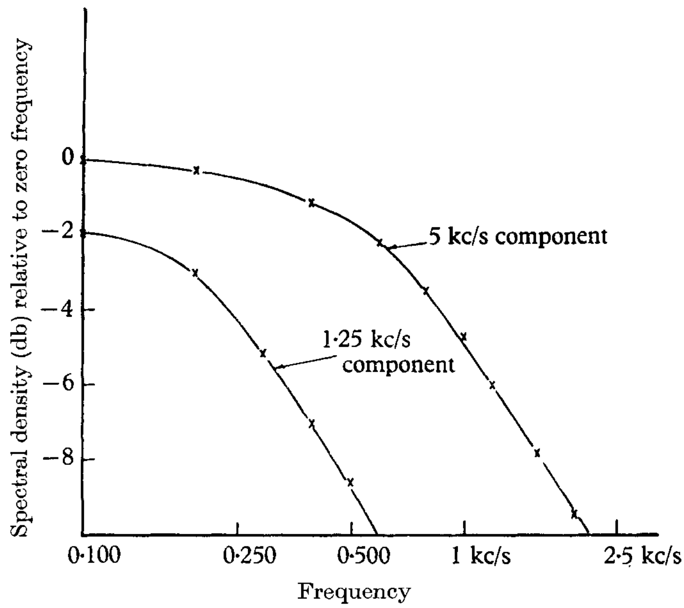

FIGURE 7. Spectra of temporal fluctuations associated with various components of the turbulent flow.

viewed in a similar frame of reference, contains only components of zero frequency, the energy of these components is spread over a large frequency range. In fact consideration of equation (3) indicates that half the energy is contained above the frequency given by $\omega_c = K$, whilst 20 % of the energy is above a frequency $\omega_c' = 3K$. For the 1250 c/s component the energy in the moving frame is thus evenly distributed about a frequency $f = 200$ c/s whilst for the 5 kc/s fixed-axis component the equivalent figure is 700 c/s. We see therefore that in both cases for half the energy measured in these frequency bands an uncertainty exists in the wave-number of the order of 15 % whilst for 20 % of the energy this uncertainty rises to almost 45 %. It is further of interest to notice that the figure of 15 % is of an order consistent with the value of $\overline{|u-\overline{u}|}/\overline{u}$ of the order of 19 % found in this region of the jet, indicating that it is fluctuations of velocities about the convection velocity that are responsible for the temporal fluctuations.

2.5. *The variation of convection velocity*

We have observed in figure 4 that the measured convection velocity is a function of frequency rising by some 30 % between 500 c/s and 5 kc/s. This would suggest that in general the higher-frequency components contain a rather higher percentage of the faster moving eddies than do those of lower frequency. Thus to some extent an eddy may appear as a high frequency because of its excessive velocity rather than its high wave-number.

A further feature worthy of mention at this time is the variation of convection, as measured using the unfiltered signal, with position across the mixing region previously reported by the present authors (1963). These results have indicated clearly that the convection velocity is higher than the mean velocity in the outer half of the mixing region, the converse being true for the inner

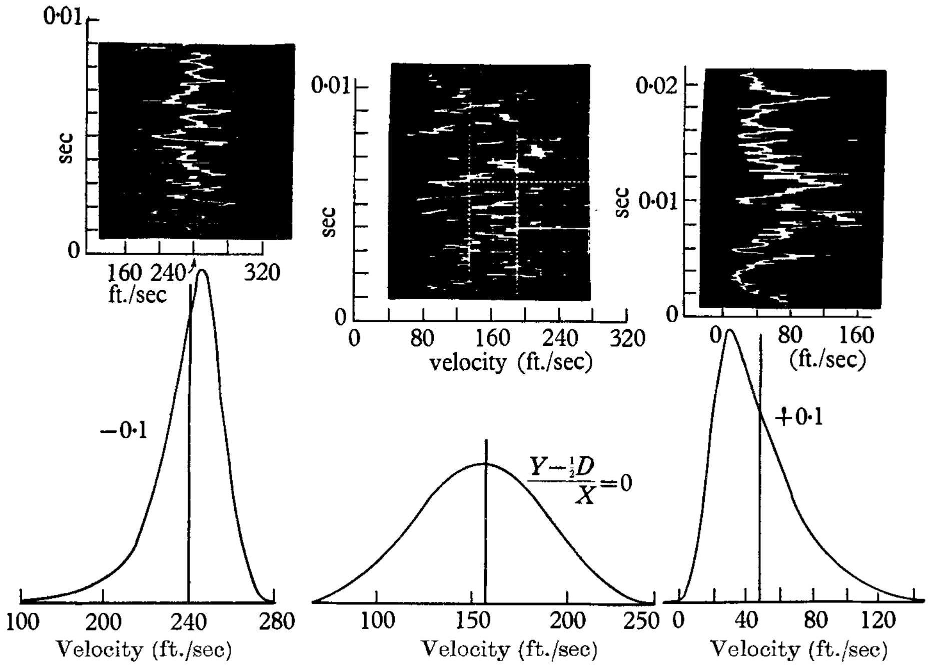

FIGURE 8. Probability density of the velocity fluctuations.

portion. Subsequent experiments, Davies & Fisher (1963), to measure the probability distribution of the velocity fluctuations indicate that a positive skewness exists on the outer half of the mixing region, whilst a negative value is found towards the potential cone (see figure 8). The mean value of the fluctuations is defined such that if v denotes the magnitude of a fluctuation relative to the mean then $\int v\,dt$ is zero. Thus if the velocity makes an appreciable number of large positive excursions, as is indicated by a positive value of the skewness, these excursions must exist for a shorter time than those smaller excursions in the negative direction in order that $\int v\,dt$ should remain zero. Conversely in the region of negative skewness the negative excursions must exist for a relatively shorter time than the smaller positive fluctuations. The results of the convection velocity measurements, however, indicate that the measured values are strongly influenced by these large fluctuations in spite of the short time for which they are present.

Consideration of the definition of the cross-correlation coefficient (§1.1) shows that this is the effect to be expected. Consider two velocity fluctuations v_1

and v_2, v_1 being a large positive excursion existing for time δt_1, whilst v_2 is a smaller negative fluctuation existing for a longer time δt_2 such that

$$v_1 \delta t_1 + v_2 \delta t_2 = 0.$$

If now it is assumed that both v_1 and v_2 reach the second measuring position undistorted then their respective contributions to the covariance will be $v_1^2 \delta t_1$ and $v_2^2 \delta t_2$. Thus it is the large excursion component which makes the larger contribution to the total covariance the difference being

$$v_1^2 \delta t_1 - v_2^2 \delta t_2 = v_2^2 \delta t_2 [(\delta t_2/\delta t_1) - 1].$$

In the case considered the larger excursion component is travelling with the greater velocity. Thus the position of the cross-correlation curve in time, from which the convection velocity is estimated will be largely controlled by this velocity. It seems probable therefore that this effect is the cause of the observed difference between the mean velocity and convection velocity across the mixing region. It is also of interest to note that it is in the region of almost zero skewness, at the centre of the mixing region, that the two velocities are most nearly equal.

We have seen above that it is the 'squaring' process which causes the large fluctuations to have a higher contribution in spite of the fact that they are of shorter duration than the small amplitude components. A similar squaring process is undertaken when obtaining an energy spectrum. Therefore once again it is the large amplitude fluctuations which will contribute most to the spectrum. These are, however, moving at velocities relatively far removed from the mean velocity so that, even ignoring errors due to temporal fluctuations, any attempt to transform these measurements to obtain space scales using the mean velocity will be in error by an amount which will be a function of the skewness.

3. Conclusions

In the course of the present paper we have seen that the presence of mean shear and an appreciable turbulent intensity in the mixing region of a jet make the interpretation of turbulence measurements considerably less clear cut than would be the case for a frozen pattern. In particular the transformation of measurements made at a single point, as a function of time, to space measurements is made sufficiently uncertain for the present authors to advocate the use of two-point space measurement wherever possible. Although most of the present experience has been gained in the mixing region of a jet, there seems no reason why similar difficulties should not be experienced in other fields of turbulence measurement, where the values of shear and intensity are similarly appreciable.

The results of measuring energy spectra of the various fixed-axes frequency components in frames of reference moving with their respective convection velocities has shown that the energy is spread over a considerable band of frequencies. With the assumption of an exponential form for the moving-axes auto-correlations this energy is evenly distributed about a radian frequency which is the inverse of the moving-axes time scale. Using the experimentally determined values of this time scale this frequency has been shown to be of the order of 15 % of the fixed-axes frequency, a value suggesting that these temporal fluctuations may, to a large extent, be due to the range of velocities about the convection velocity. An interesting experiment to investigate this effect further would be the measurement of the probability distributions of the various fixed-axes frequency components. A comparison of their various flatness factors would indicate whether the low spectral density of the higher-frequency components was due to the fact that these were small amplitude fluctuations, or whether they are in fact larger amplitude components of a more intermittent nature as is suggested by their moving-axes spectra. Considerable care would be necessary in such an experiment to ensure that the filter edges were sufficiently sharp to ensure that attenuation of signals with frequencies near the band edges did not appreciably alter results.

The observed differences between the mean velocity and convection velocity have been shown to be due to the skewness of the probability distribution of the velocity fluctuations. A useful piece of theoretical work at this time would be the derivation of a relationship between this difference and the value of the skewness. In the present experiment, working in a region of almost zero skewness, it has been demonstrated that the convection velocity increases with increased frequency suggesting to some extent that the high-frequency components are due to the high-velocity eddies rather than large wave-number components. Further information relating to this point could be obtained from the skewness of the probability distributions measured as suggested above. For example, a negative value of skewness for the low-frequency components and a positive value for the high-frequency components would suggest that very little correlation exists between wave-numbers and fixed-axis frequency. It is unfortunate that in performing such an experiment the skewness would be measured relative to the mean value of the particular component rather than relative to the mean value of the total signal. This means that the absolute value of the velocity would be lost. However, it is felt that, in spite of this disadvantage, some useful qualitative information could be obtained.

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